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ON r*bg* - CLOSED MAPS AND r*bg* - OPEN MAPS IN TOPOLOGICAL SPACES

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ABSTRACT
The purpose of this paper is to introduce r*bg* - closed maps and r*bg* - open maps and study their behaviour and properties in topological spaces. Additionally we discuss some relationships between r*bg* - closed maps and other existing closed maps. Moreover we investigate and obtain some interesting theorems.

KEYWORDS: closed maps, open maps, r*bg* - closed maps, r*bg* - open maps, g – closed maps, b* - closed maps.

INTRODUCTION
Levine **[9]** introduced the concept of generalized closed sets in topological spaces. Andrijevic **[2]** introduced and studied the concepts of b-open sets. Palaniappan et.al **[16]** introduced and investigate the concept of regular generalized closed set. Elakkiya et. al **[8]** introduced and studied r*bg* - closed sets. Generalized closed mappings were introduced and studied by Malghan **[12]**. rg - closed maps were introduced and studied by Arokiarani **[3]** . A.A.Omari and M.S.M. Noorani **[1]** introduced and studied b-closed maps. Muthuvel et.al **[14]** introduced and studied b* - closed maps in topological spaces.

In this paper, we introduce r*bg* - closed maps and r*bg* - open maps in topological spaces and investigate some of their properties.

2 Preliminaries
Definition 2.1: A subset A of a topological space (X, τ) is said to be

i. Generalized closed (briefly g - closed) set **[9]** if cl(A) ⊆ U whenever A ⊆ U and U is open in X.
ii. semi - generalised closed (briefly sg - closed) set **[5]** if scl(A) ⊆ U whenever A ⊆ U and U is semi open in X.
iii. generalized semi closed (briefly gs - closed) set **[4]** if scl(A) ⊆ U whenever A ⊆ U and U is open in X.
iv. generalized α – closed (briefly gα - closed) set **[11]** if αcl(A) ⊆ U whenever A ⊆ U and U is α – open in X.
v. semi – weakly generalized closed (briefly swg – closed) set **[15]** if cl(int(A)) ⊆ U whenever A ⊆ U and U is semi open.
vi. regular generalized closed (briefly rg - closed) set **[16]** if cl(A) ⊆ U whenever A ⊆ U and U is regular open in X.
vii. α – generalized regular closed (briefly αgr - closed) set **[18]** if αcl(A) ⊆ U whenever A ⊆ U and U is regular open in X.
viii. b* - closed set **[13]** if int(cl(A)) ⊆ U whenever A ⊆ U and U is b – open in X.
ix. g*s – closed set **[17]** if scl(A) ⊆ U whenever A ⊆ U and U is gs – open in X.
x. r*bg* - closed set **[8]** if rbcl(A) ⊆ U whenever A ⊆U and U is b – open in X.
xi. b – closed set **[2]** if cl(int(A)) ∩ int(cl(A)) ⊆ A.

Definition 2.2 :
Let X and Y be two topological spaces. A map f : (X, τ) → (Y, σ) is called

i. g - closed map **[12]** if the image of every closed set in (X, τ) is g - closed in (Y, σ).
ii. b - closed map **[1]** if the image of every closed set in (X, τ) is b - closed in (Y, σ).
iii. sg - closed map **[7]** if the image of every closed set in (X, τ) is sg - closed in (Y, σ).
iv. gs - closed map **[6]** if the image of every closed set in (X, τ) is gs - closed in (Y, σ).
v. gα - closed map **[10]** if the image of every closed set in (X, τ) is gα - closed in (Y, σ).
vi. swg - closed map **[15]** if the image of every closed set in (X, τ) is swg - closed in (Y, σ).

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vii. rg - closed map **[3]** if the image of every closed set in (X, τ) is rg - closed in (Y, σ).

viii. αgr - closed map **[18]** if the image of every closed set in (X, τ) is αgr - closed in (Y, σ).

ix. b* - closed map **[14]** if the image of every closed set in (X, τ) is b* - closed in (Y, σ).

x. g*s - closed map **[17]** if the image of every closed set in (X, τ) is g*s - closed in (Y, σ).

3 r*bg* - Closed Maps and r*bg* - Open Maps

Definition 3.1 Let X and Y be two topological spaces. A map f : (X, τ) → (Y, σ) is called r*bg* - closed map if the image of every closed set in (X, τ) is r*bg* - closed in (Y, σ).

Example 3.2

Let X = Y = {a, b c} with topologies τ = {φ, X, {a}} and σ = {φ, Y, {b}}. Let f : X → Y be defined by f(a) = b, f(b) = a, f(c) = c. Then f is r*bg* - closed map.

Definition 3.3

Let X and Y be two topological spaces. A map f : (X, τ) → (Y, σ) is called r*bg* - open map if the image of every open set in (X, τ) is r*bg* - open in (Y, σ).

Example 3.4

Let X = Y = {a, b c} with topologies τ = {φ, X, {a}} and σ = {φ, Y, {a}, {b, c}}. Let f : X → Y be the identity map. Then f is r*bg* - open map.

Theorem 3.5

If f : X → Y be a function from a topological space (X, τ) into a topological space (Y, σ) is an closed map then it is r*bg* - closed map but not conversely.

Proof : Let f : X → Y be an closed map. Let F be any closed set in X. Then f (F) is an closed set in Y. Since every regular closed set is r*bg* - closed set and every regular closed is closed, f(F) is a r*bg* - closed set. Therefore f is r*bg* - closed map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.6

Let X = Y = {a, b c} with topologies τ = {φ, X, {b}} and σ = {φ, Y, {c}}. Let f : X → Y be the identity map. Then f is r*bg* - closed but not closed. Since the image of the closed set {a, c} in X is {a, c} is not closed in Y.

Theorem 3.7

Every r*bg*- closed map is b – closed map but not conversely.

Proof:

Let f : X → Y be r*bg* - closed map and V be an closed set in X then f (V) is r*bg* - closed in Y. Since every r*bg* - closed set is b – closed set, f(V) is a b – closed set in Y. Then f is b – closed map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.8

Consider X = Y = {a, b, c} with topologies τ ={φ, X,{a}} and ={φ, Y, {c},{a, c}}. Let f : X → Y be an identity map. Then f is b – closed map but not r*bg* - closed map, since for the closed set {b, c} in X, f({b, c}) = {b, c} is not r*bg* - closed in Y.

Theorem 3.9

Every r*bg*- closed map is g – closed map but not conversely.

Proof:

Let f : X → Y be r*bg* - closed map and V be an closed set in X then f (V) is r*bg* - closed in Y. Since every r*bg* - closed set is g – closed, f(V) is g - closed in Y. Then f is g – closed map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.10

Consider X = Y = {a, b, c} with topologies τ ={φ, X,{a}} and ={φ, Y, {b}}. Let f : X → Y be an identity map. Then f is g – closed map but not r*bg* - closed map, since for the closed set {b, c} in X, f({b, c}) = {b, c} is not r*bg* - closed in Y.

Theorem 3.11

Every r*bg*- closed map is gs – closed map but not conversely.

Proof:

Let f : X → Y be r*bg* - closed map and V be an closed set in X then f (V) is r*bg* - closed in Y. Since every r*bg* - closed set is gs – closed, f(V) is gs - closed in Y. Then f is gs – closed map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.12

Consider X = Y = {a, b, c} with topologies τ ={φ, X,{a}, {a, b}} and ={φ, Y, {b},{a, b}, {b, c}}. Let f : X → Y be an identity map. Then f is gs – closed map but not r*bg*-closed map, since for the closed set {b, c} in X, f({b, c}) = {b, c} is not r*bg* - closed in Y.

Theorem 3.13

Every r*bg*- closed map is gα – closed map but not conversely.

Proof:

Let f : X → Y be r*bg* - closed map and V be an closed set in X then f (V) is r*bg* - closed in Y. Since every r*bg* - closed set is gα – closed, f(V) is gα - closed in Y. Then f is gα – closed map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.14

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Consider X = Y = {a, b, c} with topologies τ ={φ, X,{c}, {a, c}} and ={φ, Y, {a},{a, b}}. Let f : X → Y be defined by f(a) = c, f(b) = b, f(c) = a. Then f is gα – closed map but not r*bg*-closed map, since for the closed set {b} in X, f({b}) = {b} is not r*bg* - closed in Y.

Theorem 3.15

Every r*bg*- closed map is swg - closed but not conversely.

Proof:

Let f : X → Y be r*bg* - closed map and V be an closed set in X then f (V) is r*bg* - closed in Y. Since every r*bg* - closed set is swg – closed, f(V) is swg - closed in Y. Then f is swg – closed map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.16

Consider X = Y = {a, b, c} with topologies τ ={φ, X, {b}, {a, b}, {b, c}} and σ ={φ, Y, {a}, {b, c}}. Let f : X → Y be an identity map. Then f is swg – closed map but not r*bg* - closed map, since for the closed set {c} in X, f({c}) = {c} is not r*bg* - closed in Y.

Theorem 3.17

Every r*bg* - closed map is rg – closed map but not conversely.

Proof:

Let f : X → Y be r*bg* - closed map and V be an closed set in X then f (V) is r*bg* - closed in Y. Since every r*bg* – closed is rg - closed in Y, f(V) is rg – closed in Y. Then f is rg – closed map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.18

Consider X = Y = {a, b, c} with topologies τ ={φ, X, {b}, {a, b}, {b, c}} and σ ={φ, Y, {a}, {a, b}}. Let f : X → Y be an identity map. Then f is rg – closed map but not r*bg*-closed map, since for the closed set {a, c} in X, f({a, c}) = {a, c} is not r*bg* - closed in Y.

Theorem 3.19

Every r*bg*- closed map is αgr – closed map but not conversely.

Proof:

Let f : X → Y be r*bg* - closed map and V be an closed set in X then f (V) is r*bg* - closed in Y. Since every r*bg* - closed set is αgr – closed, f(V) is αgr – closed in Y. Then f is αgr – closed map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.20

Consider X = Y = {a, b, c} with topologies τ ={φ, X, {b}, {a, c}} and σ ={φ, Y, {a},{a, b}}. Let f : X → Y be an identity map. Then f is αgr – closed map but not r*bg*-closed map, since for the closed set {b} in X, f({b}) = {b} is not r*bg* - closed in Y.

Remark 3.21

The following examples show that r*bg* - closed maps are independent of sg – closed maps, g*s - closed maps.

Example 3.22

Let X = Y = {a, b, c} with the topologies τ = {φ, X, {c}, {a, b}} and σ = {φ, Y, {a}, {b},{a, b}}. Let f : X → Y be defined by f(a) = a, f(b) = c, f(c) = b. Then f is sg – closed map but not r*bg* - closed map.

Example 3.23

Let X = Y = {a, b, c} with topologies τ = {φ, X, {a}, {b}, {a, b}} and σ = {φ, Y, {b}}. Let f : X → Y be defined by f(a) = a, f(b) = c, f(c) = b. Then f is r*bg* - closed map but not sg – closed map.

Example 3.24

Let X = Y = {a, b, c} with topologies τ = {φ, X, {c}} and σ = {φ, Y, {a}, {b, c}}. Let f : X → Y be an identity map. Then f is g*s – closed map but not r*bg* - closed map.

Example 3.25

Let X = Y = {a, b, c} with topologies τ = {φ, X, {a}} and σ = {φ, Y, {c}}. Let f : X → Y be defined by f(a) = c, f(b) = a, f(c) = b. Then f is r*bg* - closed map but not g*s – closed map.

Theorem 3.26

Every open map in (X, τ) is r*bg* - open map but not conversely.

Proof :

Let f : X → Y be an open map. Let F be any open set in X. Then f(F) is an open set in Y. Since every regular open set is r*bg* - open and every regular open is open, f(F) is a r*bg* - open set. Therefore f is r*bg* - open map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.27

Let X = Y = {a, b, c} with topologies τ = {φ, X , {c}} and σ = {φ, Y, {a}, {b}, {a, b}, {b, c}}. Let f : X → Y be the identity map. Then f is r*bg* - open map but not an open map, Since the image of the open set {c} is not open in Y.

Remark 4.33

The following diagram shows the relationship between r*bg* - closed maps with various existing closed maps.

In this diagram CM means closed maps.

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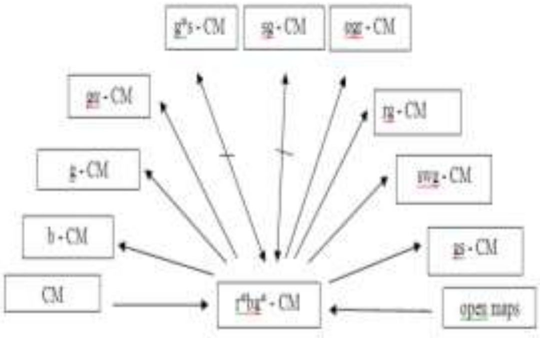

Theorem 3.28

A map f : X → Y is r*bg* - closed if and only if for each subset S of Y and for each open set U containing f^{-1} (S) there exists a r*bg* - open set V of Y such that S ⊆ V and f^{-1}(V) ⊆ U.

Proof :

Necessity: Suppose that f is a r*bg* - closed map. Let S ⊆ Y and U be an open subset of X such that f^{-1}(S) ⊆ U. Let V = Y - f(X - U). Since f is r*bg* - closed, V is r*bg* - open set containing S such that f^{-1}(V) ⊆ U.

Sufficiency: let F be a closed set of X. Then $f^{-1}((f(F))^c) \subseteq F^c$ and F^c is open. By assumption, there exist a r*bg* - open set V of Y such that $(f(F))^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$ and so $F \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(F) \subseteq f((f^{-1}(V))^c) \subseteq V^c$ which implies that $f(F) = V^c$. Since V^c is r*bg* - closed, f(V) is r*bg* - closed and therefore f is r*bg* - closed.

Theorem 3.29

If f : X → Y is a closed map and g : Y → Z is r*bg* - closed map then g ∘ f : X → Z is r*bg* - closed map.

Proof :

Let f : X → Y is a closed map and g : Y → Z is a r*bg* - closed map. Let V be any closed set in X. Since f : X → Y is closed map, f(V) is closed in Y and since g : Y → Z is r*bg* - closed map, g(f(V)) is r*bg* closed in Z. Therefore g ∘ f : X → Z is r*bg* - closed map.

Remark 3.30

The following example shows that the composition of two r*bg* - closed maps is r*bg* - closed map.

Example 3.31

Let X = Y = {a, b, c} with topologies τ = {φ, X, {a}}, σ = {φ, Y, {b}} and η = {φ, Z, {a}, {b, c}}. Define a map f : X → Y by f(a) = b, f(b) = a and f(c) = c and a map g : Y → Z by g(a) = b, g(b) = a, g(c) = c. Then both f and g are r*bg* - closed maps but their composition g ∘ f : X → Z is not r*bg* - closed map. Since for the closed set {b, c} in X, (g ∘ f)({b, c}) = {b, c} is r*bg* - closed set in Z.

Remark 3.32

If f : X → Y is r*bg* closed map and g : Y → Z is closed then their composition need not be a r*bg* - closed map as seen from the following example.

Example 3.33

Let X = Y = {a, b, c} with the topologies τ = {φ, X, {a}, {b}, {a, b}, {b, c}}, σ = {φ, Y, {a, b}, {b}} and η = {φ, Z, {b}, {b, c}, {a, b}}. Define a map f : X → Y by f(a) = a, f(b) = c, f(c) = b and g : Y → Z by g(a) = b, g(b) = a, g(c) = c. Then f is a r*bg* - closed map and g is closed map. But their composition g ∘ f : X → Z is not a r*bg* - closed map. Since for the closed set {a, b} in X, (g ∘ f)({a, b}) = {b, c} is not r*bg* - closed in Z.

Theorem 3.34

For any bijection f : (X, τ) → (Y, σ) the following are equivalent.

a) f^{-1} : (Y, σ) → (X, τ) is r*bg* – continuous.
b) f is a r*bg* – open map.
c) f is a r*bg* – closed map.

Proof:

a) ⇒ (b) : Let U be an open set of (X, τ). By assumption, $(f^{-1})^{-1}(U)$ = f (U) is r*bg* – open in (Y, σ) and so f is r*bg* – open map.
b) ⇒ (c) : Let F be a closed set of (X, τ). Then F^c is open in (X, τ). By assumption, f (F^c) is r*bg* – open in (Y, σ). (i.e), $f(F^c) = (f(F))^c$ is r*bg* – open in (Y, σ) and therefore f (F) is r*bg* – closed in (Y, σ). Hence f is r*bg* – closed.
c) ⇒ (a) : Let F be a closed set of (X, τ). By assumption, f (F) is r*bg* –closed in (Y, σ). But $f(F) = (f^{-1})^{-1}(F)$ and therefore f^{-1} is r*bg* –continuous on Y.

Theorem 3.35

If f : X → Y is a continuous, r*bg* - closed map from a normal space X onto a space Y, then Y is normal.

Proof :

Let A, B be disjoint closed sets of Y. Then f^{-1}(A) and f^{-1}(B) are disjoint closed set of X. Since X is normal there are disjoint open sets U, V in X such that f^{-1}(A) ⊆ U and f^{-1}(B) ⊆ V. Since f is r*bg* - closed, then by theorem 3.28, there exists r*bg* - open sets G, H in Y such that A ⊆ G, B ⊆ H, f^{-1}(G) ⊆ U and f^{-1}(H) ⊆ V. Since U, V are disjoint, int(G) and int(H) are disjoint open sets. Since G is

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r*bg* - open A is closed and A ⊆ G, A ⊆ int(G). Similarly B ⊆ int(H). Hence Y is normal.

Theorem 3.48

If f : (X, τ) → (Y, σ) is an open, continuous, r*bg*- closed surjection and cl(F) = F for every r*bg* - closed set in (Y, σ), where X is regular, then Y is regular.

Proof :

Let U be an open set in Y and p ∈ U. Since f is surjection, there exists a point x ∈ X such that f(x) = p. Since X is regular and f is continuous, there is an open set V in X such that x ∈ V ⊂ cl(V) ⊂ f^{-1}(U). Here p ∈ f(V) ⊂ f(cl(V)) ⊂ U → (i).

Since f is r*bg*- closed, f(cl(V)) is r*bg*- closed set contained in the open set U. By hypothesis, cl(f(cl(V))) = f(cl(V)) and cl(f(V)) = cl(f(cl(V))) → (ii).

From (i) and (ii), we have p ∈ f(V) ⊂ cl(f(V)) ⊂ U and f(V) is open, since f is open. Hence Y is regular.

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